9. cvičení - výsledky

Příklad 1.

(a) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \begin{cases} -1, & x \in (-\infty, 0) \\ 1, & x \in (0, \infty). \end{cases}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$; v $x = 0$ je derivace zleva -1, zprava 1

(b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \begin{cases} 3 - 2x, & x \in (-\infty, 0) \cup (3/2, \infty), \\ 2x, & x \in (0, 3/2) \end{cases}$,
$f'_-(0) = 3, f'_+(0) = 0, f'_-(3/2) = 3, f'_+(3/2) = 0$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0, \frac{3}{2}\}$

(c) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = \begin{cases} 1/x, & x \in (-\infty, -1) \cup (1, \infty), \\ -1/x, & x \in (-1, 0) \cup (0, 1), \end{cases}$,
$f'_-(-1) = -1, f'_+(-1) = 1, f'_-(0) = \infty, f'_+(0) = -\infty, f'_-(1) = -1, f'_+(1) = 1$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{-1, 0, 1\}$

(d) $f'(x) = \operatorname{sgn} x$ na $\mathbb{R} \setminus \{0\}$

(e) $f' = \begin{cases} 1, & x \in (-\infty, 0], \\ 1/(1 + x), & x \in (0, \infty) \end{cases}$,
Pozor. Bod $x = 0$ je třeba vyšetřit zvláště.

Příklad 2.

(a) $\mathcal{D}_f = (0, 1) \cup (1, \infty)$; $f'(x) = -\frac{1}{x \cdot \log^2 x}$; $\mathcal{D}_{f'} = (0, 1) \cup (1, \infty)$

(b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = 9(3x^2 - 1)\cos^8(x^3 - x + 2) \cdot \left(-\sin\left(x^3 - x + 2\right)\right)$; $\mathcal{D}_{f'} = \mathbb{R}$

(c) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \cos(\sin(\sin x)) \cdot \cos(\sin x) \cdot \cos x$; $\mathcal{D}_{f'} = \mathbb{R}$

(d) $\mathcal{D}_f = (-\infty, -1) \cup (1, \infty)$; $f'(x) = \frac{8x}{(x^2-1)(x^2+1)}$; $\mathcal{D}_{f'} = (-\infty, -1) \cup (1, \infty)$

(e) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \sin(2x) - 2x\cos(x^2)$; $\mathcal{D}_{f'} = \mathbb{R}$

(f) $\mathcal{D}_f = (1, e) \cup (e, \infty)$; $f'(x) = \frac{6}{\log\left(\log^3(x)\right)} \cdot \log(x) \cdot x$; $\mathcal{D}_{f'} = (1, e) \cup (e, \infty)$

(g) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \begin{cases} 2, & x \in (-\infty, -3) \cup (1, \infty) \\ -2x, & x \in (-3, 1) \end{cases}$, $f'_+(-3) = 6, f'_-(-3) = -2, f'_+(1) = 2, f'_-(1) = -2$;
$\mathcal{D}_{f'} = \mathbb{R} \setminus \{-3, 1\}$

(h) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \begin{cases} -1, & x \in (-\infty, 1] \\ 2x - 3, & x \in (1, 2] \\ 1, & x \in (2, \infty) \end{cases}$; $\mathcal{D}_{f'} = \mathbb{R}$

(i) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \begin{cases} 2x \sin \frac{1}{\sqrt[3]{x}} - \frac{1}{3} x^{\frac{2}{3}} \cos \frac{1}{\sqrt[3]{x}}, x \neq 0, \\ 0, & x = 0. \end{cases}$; $\mathcal{D}_{f'} = \mathbb{R}$

(j) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \begin{cases} 0, & x \neq 0, \\ -\infty, & x = 0. \end{cases}$; $\mathcal{D}_{f'} = \mathbb{R}$

Příklad 3.

(a) $\frac{152}{37}$

(b) 1

(c) $\frac{1}{6}$

(d) $\frac{-1}{3}$

(e) 0

(f) 1

(g) $-\frac{1}{3}$

(h) 0

(i) 1

(j) $\frac{1}{3}$

(k) ∞

(l) $-\frac{1}{6}$

(m) 0

(n) 0

(o) 0

(p) 0

Příklad 4.

(a) $f' = \begin{cases} 1, & x \in ((2k+1)\pi, (2k+2)\pi), k \in \mathbb{Z} \\ \frac{4\cos x}{\sin^2 x + 1} + 1, & x \in (2k\pi, (2k+1)\pi), k \in \mathbb{Z} \end{cases}$
Dále pak $f'_+(2k\pi) = 5,\ f'_-(2k\pi) = 1,\ f'_+((2k+1)\pi) = 1,\ f'_-((2k+1)\pi) = -3.$

(b) $f' = \begin{cases} 0, & x \in \left(-\frac{\pi}{3} + k\pi, \frac{\pi}{3} + k\pi\right), k \in \mathbb{Z} \\ -\sin x, & x \in \left(\frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi\right), k \in \mathbb{Z} \end{cases}$

- $f'_-(-\frac{\pi}{3} + 2k\pi) = \sqrt{3}/2,\ f'_+(-\frac{\pi}{3} + 2k\pi) = 0,$
- $f'_-(\frac{\pi}{3} + 2k\pi) = 0,\ f'_+(\frac{\pi}{3} + 2k\pi) = -\sqrt{3}/2,$
- $f'_-(\frac{2\pi}{3} + 2k\pi) = -\sqrt{3}/2,\ f'_+(\frac{2\pi}{3} + 2k\pi) = 0,$
- $f'_-(\frac{4\pi}{3} + 2k\pi) = 0,\ f'_+(\frac{4\pi}{3} + 2k\pi) = \sqrt{3}/2.$

(c) $f' = \frac{2|x|}{x^3+x}, x \in \mathbb{R} \setminus \{0\},\ f'_-(0) = -2,\ f'_+(0) = 2$

(d) $f'(x) = \begin{cases} e^{x-1}(2x + x^2), & x < 1, \\ e^{1-x}(2x - x^2), & x > 1, \end{cases} f'_-(1) = 3, f'_+(1) = -1.$

(e) $f' = \frac{\sqrt{2}}{(\sin x + \cos x)^2}, x \in \mathbb{R} \setminus \{-\pi/4 + k\pi \colon k \in \mathbb{Z}\},$
A jednostranné derivace $f'_-(-\pi/4 + k\pi) = \infty,\ f'_+(-\pi/4 + k\pi) = -\infty.$

(f) $f' = \frac{xe^{-x^2}}{\sqrt{1-e^{-x^2}}}, x \in \mathbb{R} \setminus \{0\},\ f'_-(0) = -1,\ f'_+(0) = 1$

(g) $f'(x) = \frac{2x}{\sqrt{1-\left(\frac{1}{1+x^2}\right)^2 \cdot (1+x^2)^2}}, x \neq 0.$

Jednostranné derivace $f'_-(0) = -\sqrt{2}, f'_+(0) = \sqrt{2}$.

(h) $f'(x) = 2x\left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)\right) + x^2\left(-\frac{1}{x^2}\cos\left(\frac{1}{x}\right) + \frac{1}{x^2}\sin\left(\frac{1}{x}\right)\right), x \neq 0$

V nule z definice $f'(0) = \lim_{x\to 0} x(\sin\frac{1}{x} + \cos\frac{1}{x}) = 0$.

(i) $f'(x) = x^{x^2} \cdot (2x \log x + x)$.

(j) $f'(x) = \begin{cases} -\sin x, & x \in (0,1), \\ -2x \sin x^2, & x \in (-\infty, 0) \cup (1, \infty). \end{cases}$

Dále $f'(0) = 0$ a $f'_-(1) = -\sin 1,\ f'_+(1) = -2\sin 1$

(k) $f'(x) = \begin{cases} 1, & x \in (\frac{-1+\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}), \\ -2x, & x \in (0, \frac{-1+\sqrt{5}}{2}), \\ 2(x-1), & x \in (-\infty, 0) \cup (\frac{3+\sqrt{5}}{2}, \infty). \end{cases}$

Pak spočteme jednostranné derivace:

- $f'_-(\frac{-1+\sqrt{5}}{2}) = 2 - 2\sqrt{5},\ f'_+(\frac{-1+\sqrt{5}}{2}) = \frac{-1+\sqrt{5}}{2}$,
- $f'_-(\frac{3+\sqrt{5}}{2}) = 1,\ f'_+(\frac{3+\sqrt{5}}{2}) = 1 + \sqrt{5}$,
- $f'_-(0) = -2,\ f'_+(0) = 0$.

(l) $f'(x) = \begin{cases} \frac{2\tan x \frac{1}{\cos^2 x}}{1+\tan^4 x}, & x \neq \frac{\pi}{2} + k\pi \\ 0, & x = \frac{\pi}{2} + k\pi. \end{cases}$